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Heat, Light and Relativity

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Abstract

In this paper I argue that because we always observe nature through some spatial-temporal averaging operation we must interpret all observed statistical covariances of velocities and gradients as partial time derivatives. Systematic application of this result leads to a new interpretation of the radiation wave equation in which c^2 measures the statistical variance of velocity. The Galilean invariance of c^2 is then automatic. These results enable us to recast the Einstein-Minkowski space-time formalism within the framework of classical statistical mechanics. However, the Einstein work-energy relation and the constancy of the speed of light appear as equivalent approximations which become exact only in the adiabatic limit.

1. *Introduction*

Heat, being defined as a mode of energy transfer that is distinct from work, lies outside the scope of Newtonian mechanics because of the Newtonian equivalence between work and energy. However, by extending the framework of Newtonian mechanics to take account of the averaging operations which are associated with observation, one readily arrives at a statistical mechanical theory of heat (Kornacker, 1968).

It is generally recognised that heat takes the form of electromagnetic radiation in free space. Therefore, it is somewhat surprising that the fundamental electromagnetic field equations have not been given a statistical mechanical interpretation.

Two deeply rooted traditions in theoretical physics are responsible for this omission. Statistical mechanical equations are commonly associated with the concepts of 'randomness' and 'irreversibility', while field equations are commonly associated with 'determinism' and 'time-reversal symmetry'. If these associations are accepted then it is difficult to contemplate any unification of the theories of heat and light.

In this paper I show that the link between the theories of heat and light lies in the statistical theory of partial time derivatives. Partial time derivatives do not appear in the fundamental equations of Newtonian physics. The partial time derivatives of position and momentum are simply equivalent to the respective total time derivatives, and the partial time derivative of energy vanishes.

The radiation wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \sum \frac{\partial^2 \psi}{\partial x_i{}^2} \tag{1.1}$$

was derived by Maxwell from the phenomenological macroscopic laws of electric and magnetic fields and has become the basis for the electromagnetic theory of light. By accepting this equation, which attributes fundamental significance to partial time derivatives, physicists implicitly abandoned the framework of Newtonian concepts. However, it was not the appearance of $\partial/\partial t$ that caused concern; it was the appearance of a 'velocity' c whose observed value is independent of the relative velocity between the emitter and the observer (Galilean invariance).

It has generally been assumed that the Galilean invariance of c in equation (1.1) is an illusion in some sense. Several futile attempts to explain the illusion were made by Maxwell, Lorentz, and others. Einstein then axiomatised the illusion as a fundamental law governing spatial and temporal observations. Einstein introduced the concept of 'relativistic' velocity and derived the most general transformation laws that would preserve Newtonian physics in the limit of small velocities and yet make the relativistic velocity a Galilean invariant when it equalled the speed of light.

The alternative formalism which follows is based on a statistical theory of partial time derivatives. The results unify the theories of heat and light, and clarify the meaning and limitations of the Einstein formalism.

2. *Partial Time Derivatives*

All measurements include a local spatial-temporal averaging (Kornacker, 1968) operation $\langle\ \rangle$. If this fact is not expressed explicitly, then in theoretical physics we will be trying to describe the observed quantity $\langle\phi(\mathbf{x})\rangle$ as a mathematical function in the form $\phi(\langle\mathbf{x}\rangle)$. When this fails we formally introduce an explicit time dependence and require that

$$\psi(\langle\mathbf{x}\rangle, t) = \langle\phi(\mathbf{x})\rangle \tag{2.1}$$

Now suppose that the distribution function in $\langle\ \rangle$ does not change in time, which is to say that the response characteristics of the measuring devices are stable (ideal measuring devices). Then the operators $\langle\ \rangle$ and d/dt commute. Therefore we may write

$$\frac{d\langle\phi\rangle}{dt} = \left\langle\frac{d\phi}{dt}\right\rangle \tag{2.2}$$

Using equation (2.2) and the chain-rule expansion of $d\phi/dt$ then yields

$$\begin{aligned}\frac{d\langle\phi\rangle}{dt} &= \left\langle\sum \frac{\partial\phi}{\partial x_i}\frac{dx_i}{dt}\right\rangle \\ &= \sum\left\langle\frac{\partial\phi}{\partial x_i}\right\rangle\left\langle\frac{dx_i}{dt}\right\rangle + \sum \sigma^2_{\frac{\partial\phi}{\partial x_i}, \frac{dx_i}{dt}}\end{aligned} \tag{2.3}$$

where the covariance σ^2 is defined by

$$\sigma^2_{A,B} = \langle AB \rangle - \langle A \rangle \langle B \rangle \tag{2.4}$$

On the other hand we must have

$$\frac{d\langle \phi \rangle}{dt} = \frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \sum \frac{\partial \psi}{\partial \langle x_i \rangle} \frac{d\langle x_i \rangle}{dt} \tag{2.5}$$

Comparing terms in equation (2.5) and (2.3) we identify

$$\frac{\partial \psi}{\partial \langle x_i \rangle} = \left\langle \frac{\partial \phi}{\partial x_i} \right\rangle \tag{2.6}$$

$$\frac{\partial \psi}{\partial t} = \sum \sigma^2_{\frac{\partial \phi}{\partial x_i}, \frac{dx_i}{dt}} \tag{2.7}$$

Equation (2.7) shows how to convert the chain-rule expression for $d\phi/dt$ into an expression for $\partial\langle \phi \rangle/\partial t$.

Having now developed a general Newtonian model for the $\partial/\partial t$ operator we are prepared to consider the form of $\partial^2/\partial t^2$. Applying $\partial/\partial t$ to each averaged term in $\partial\psi/\partial t$ gives

$$\frac{\partial^2 \psi}{\partial t^2} = \sum \sigma^2_{\frac{\partial \phi}{\partial x_i}, \frac{d^2 x_i}{dt^2}} + \sum\sum \sigma^2_{\frac{\partial^2 \phi}{\partial x_i \partial x_j}, \frac{dx_i}{dt}, \frac{dx_j}{dt}} - \sum\sum \left\langle \frac{dx_j}{dt} \right\rangle \sigma^2_{\frac{\partial^2 \phi}{\partial x_i \partial x_j}, \frac{dx_i}{dt}} \tag{2.8}$$

We now use the identity

$$\sigma^2_{A,BC} - \langle B \rangle \sigma^2_{A,C} = \langle C \rangle \sigma^2_{A,B} + \sigma^3_{A,B,C} \tag{2.9}$$

where

$$\sigma^3_{A,B,C} = \langle (A - \langle A \rangle)(B - \langle B \rangle)(C - \langle C \rangle) \rangle \tag{2.10}$$

and let

$$A = \frac{dx_i}{dt}$$

$$B = \frac{dx_j}{dt}$$

$$C = \frac{\partial^2 \phi}{\partial x_i \partial x_j}$$

to obtain the universal Newtonian wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = \sum\sum \frac{\partial^2 \psi}{\partial \langle x_i \rangle \partial \langle x_j \rangle} \sigma^2_{\frac{dx_i}{dt}, \frac{dx_j}{dt}} + \sum\sum \sigma^3_{\frac{dx_i}{dt}, \frac{\partial^2 \phi}{\partial x_i \partial x_j}, \frac{dx_j}{dt}} + \sum \sigma^2_{\frac{\partial \phi}{\partial x_i}, \frac{d^2 x_i}{dt^2}} \tag{2.11}$$

This equation reduces to equation (1.1) if for all i, j

$$\sigma^3_{\frac{dx_i}{dt}, \frac{\partial^2\phi}{\partial x_i \partial x_j}, \frac{dx_j}{dt}} = 0 \tag{2.12}$$

$$\sigma^2_{\frac{\partial\phi}{\partial x_i}, \frac{d^2x_i}{dt^2}} = 0 \tag{2.13}$$

$$\sigma^2_{\frac{dx_i}{dt}, \frac{dx_i}{dt}} = c^2 \tag{2.14}$$

and for $i \neq j$

$$\sigma^2_{\frac{dx_i}{dt}, \frac{dx_j}{dt}} = 0 \tag{2.15}$$

A plausible physical interpretation of conditions (2.12) through (2.15) could be based on the fact that the center of mass of the earth must be performing 'Brownian motion' with respect to our laboratory coordinates. Evidently c^2 measures the thermally generated variance of our coordinate velocity relative to inertial coordinates. If comparable high-frequency fluctuations are produced by local fields, as for bound electrons, then conditions (2.12) and (2.13) may break down. Conditions (2.14) and (2.15) will generally fail if either of the previous two conditions fails. The significance of non-zero dc^2/dt will be discussed at the end of this paper.

The interpretation of c as an ordinary wave velocity is based on a different set of conditions. For arbitrary $\psi(\mathbf{x}, t)$ the velocity components dx_i/dt of points of constant ψ must satisfy

$$0 = \frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \sum \frac{\partial\psi}{\partial x_i}\frac{dx_i}{dt} \tag{2.16}$$

Therefore, on any trajectory (line along which ψ is constant)

$$\begin{aligned} \frac{\partial^2\psi}{\partial t^2} &= -\sum \frac{\partial\psi}{\partial x_i}\frac{d^2x_i}{dt^2} - \sum \frac{\partial^2\psi}{\partial t\,\partial x_i}\frac{dx_i}{dt} \\ &= -\sum \frac{\partial\psi}{\partial x_i}\frac{d^2x_i}{dt^2} - \sum\sum \frac{\partial^2\psi}{\partial x_i\,\partial x_j}\frac{dx_i}{dt}\frac{dx_j}{dt} \end{aligned} \tag{2.17}$$

A first step in reducing equation (2.17) to equation (1.1) is the condition that for all i

$$\frac{d^2x_i}{dt^2} = 0 \tag{2.18}$$

Next, it is essential that

$$\sum\sum \frac{\partial^2\psi}{\partial x_i\,\partial x_j}\frac{dx_i}{dt}\frac{dx_j}{dt} = \sum \left(\frac{dx_i}{dt}\right)^2 \sum \frac{\partial^2\psi}{\partial x_j^2} \tag{2.19}$$

in which case equation (2.17) reduces to equation (1.1), with

$$c^2 = \sum \left(\frac{dx_i}{dt}\right)^2 \tag{2.20}$$

A sufficient condition to assure equation (2.19) is

$$\frac{\partial^2 \psi}{\partial x_i \, \partial x_j} \frac{dx_i}{dt} \frac{dx_j}{dt} = \frac{\partial^2 \psi}{\partial x_j^2} \left(\frac{dx_i}{dt}\right)^2 \tag{2.21}$$

By permutation of i and j condition (2.21) implies

$$\frac{\partial^2 \psi}{\partial x_i \, \partial x_j} \frac{dx_i}{dt} \frac{dx_j}{dt} = \frac{\partial^2 \psi}{\partial x_i^2} \left(\frac{dx_j}{dt}\right)^2 \tag{2.22}$$

Multiplying equations (2.21) and (2.22) together then yields the sufficient condition

$$\left(\frac{\partial^2 \psi}{\partial x_i \, \partial x_j}\right)^2 = \frac{\partial^2 \psi}{\partial x_i^2} \frac{\partial^2 \psi}{\partial x_j^2} \tag{2.23}$$

Condition (2.23) is satisfied, for example, by any plane wave. Note that conditions (2.18) and (2.20) preclude a time-dependent c.

3. *Decomposition of Velocity Fluctuations*

We have found through equation (2.7) that explicit time dependence may be created by covariant fluctuations. The relativistic space-time of Minkowski is one way of describing this kind of time dependence, but before we can see this we must reformulate the statistical theory so that it parallels the standard theory.

A necessary first step is to decompose the velocity fluctuation vector

$$\boldsymbol{\delta} = \mathbf{v} - \langle \mathbf{v} \rangle \tag{3.1}$$

into the two orthogonal vectors

$$\boldsymbol{\delta}_{\parallel} = \frac{(\boldsymbol{\delta} \cdot \langle \mathbf{v} \rangle) \langle \mathbf{v} \rangle}{\langle v \rangle^2} \tag{3.2}$$

$$\boldsymbol{\delta}_{\perp} = \frac{\langle \mathbf{v} \rangle \times \boldsymbol{\delta} \times \langle \mathbf{v} \rangle}{\langle v \rangle^2} \tag{3.3}$$

$$\delta^2 = \delta_{\parallel}^2 + \delta_{\perp}^2 \tag{3.4}$$

The vector $\boldsymbol{\delta}_{\parallel}$ points parallel to $\langle \mathbf{v} \rangle$ while $\boldsymbol{\delta}_{\perp}$ points perpendicular to $\langle \mathbf{v} \rangle$. We then define a new velocity vector

$$\mathbf{u} = \langle \mathbf{v} \rangle + \boldsymbol{\delta}_{\parallel} \tag{3.5}$$

Clearly

$$\langle \mathbf{u} \rangle = \langle \mathbf{v} \rangle \tag{3.6}$$

and the fluctuations of $\mathbf{u}$ always point parallel to $\langle \mathbf{u} \rangle$.

The importance of **u** lies in the equations

$$\sum \sigma^2_{u_i, u_i} = \langle \delta_{\parallel}{}^2 \rangle = c^2 \tag{3.7}$$

and

$$(\sigma^2_{u_i, u_j})^2 = \sigma^2_{u_i, u_i} \sigma^2_{u_j, u_j} \tag{3.8}$$

The second equality of equation (3.7) is based on conditions (2.14) and (2.15); we are here assuming that the variance of any velocity component (e.g. the component of **v** in the direction of $\langle \mathbf{v} \rangle$) is equal to c^2 regardless of direction. This condition may, of course, break down as mentioned above.

Equation (3.8) is just another way of saying that u_i and u_j are linearly related by a non-fluctuating factor, since equation (3.8) says directly that the magnitude of the linear correlation coefficient between u_i and u_j is 1. The linear relation comes about because the fluctuations of **u** change its length but not its direction.

Now define the new spatial coordinates y_i by the equations

$$\langle y_i \rangle = \langle x_i \rangle \tag{3.9}$$

$$\frac{dy_i}{dt} = u_i \tag{3.10}$$

We may then rewrite equation (2.11) in these new coordinates by substituting y_i for x_i throughout. We may furthermore consider the special case where this substitution leaves conditions (2.12) and (2.13) valid. Then we have

$$\frac{\partial^2 \psi}{\partial t^2} = \sum \sum \frac{\partial^2 \psi}{\partial \langle x_i \rangle \, \partial \langle x_j \rangle} \sigma^2_{u_i, u_j} \tag{3.11}$$

Conditions (2.14) and (2.15) are, of course, replaced by equations (3.7) and (3.8).

The reduction of equation (3.11) to the form of equation (1.1) now requires that

$$\sum \sum \frac{\partial^2 \psi}{\partial \langle x_i \rangle \, \partial \langle x_j \rangle} \sigma^2_{u_i, u_j} = \sum \frac{\partial^2 \psi}{\partial \langle x_i \rangle^2} \sum \sigma^2_{u_j, u_j} \tag{3.12}$$

which is the analog of equation (2.19). A sufficient condition is

$$\frac{\partial^2 \psi}{\partial \langle x_i \rangle \, \partial \langle x_j \rangle} \sigma^2_{u_i, u_j} = \frac{\partial^2 \psi}{\partial \langle x_i \rangle^2} \sigma^2_{u_j, u_j} \tag{3.13}$$

and

$$\frac{\partial^2 \psi}{\partial \langle x_i \rangle \, \partial \langle x_j \rangle} \sigma^2_{u_i, u_j} = \frac{\partial^2 \psi}{\partial \langle x_j \rangle^2} \sigma^2_{u_j, u_j} \tag{3.14}$$

where equation (3.14) follows from equation (3.13) by permutation of i and j. Multiplying equations (3.13) and (3.14) together, and using equation (3.8), we obtain the sufficient condition

$$\left(\frac{\partial^2 \psi}{\partial\langle x_i\rangle\,\partial\langle x_j\rangle}\right)^2 = \frac{\partial^2 \psi}{\partial\langle x_i\rangle^2}\frac{\partial^2 \psi}{\partial\langle x_j\rangle^2} \tag{3.15}$$

which is the direct analog of equation (2.23).

4. *Special Relativity*

Based on equations (3.6) and (3.7) we now define

$$c_r^2 = \langle u_r^2\rangle - \langle v_r\rangle^2 \tag{4.1}$$

$$c^2 = c_r\,c_r \tag{4.2}$$

Here we have adopted the Einstein summation convention of adding over subscripts which are explicitly repeated in a product. We can then define a unit 4-vector by the equations

$$\alpha_r^2 = \frac{\langle v_r\rangle^2}{c^2} \tag{4.3}$$

$$\alpha_4^2 = -\frac{\langle u_r\,u_r\rangle}{c^2} \tag{4.4}$$

so that

$$\alpha_\rho\,\alpha_\rho = \frac{\langle v_r\rangle\langle v_r\rangle - \langle u_r\,u_r\rangle}{c^2} = -1 \tag{4.5}$$

We follow the standard convention that Roman subscripts are restricted to three space coordinates while Greek subscripts include all four coordinates. Introducing an imaginary fourth coordinate through equation (4.4) is merely a device enabling us to use the speed of light as a normalizing factor for the velocity vector.

Now we can write the classical statistical velocity vector as

$$\frac{d\xi_r}{dt} = c\alpha_r = \langle v_r\rangle \tag{4.6}$$

We then define

$$\frac{d\xi_4}{dt} = c\alpha_4 \tag{4.7}$$

so that $\boldsymbol{\alpha}$ sets the direction of the velocity 4-vector defined by

$$\frac{d\xi_\rho}{dt} = c\alpha_\rho \tag{4.8}$$

The classical statistical momentum vector is

$$\pi_r = m\langle v_r\rangle = m\frac{d\xi_r}{dt} \tag{4.9}$$

This may be extended to a 4-vector with

$$\pi_4 = m\frac{d\xi_4}{dt} \tag{4.10}$$

Using equations (4.4) and (4.7), equation (4.10) becomes

$$\pi_4 = i\frac{E}{c} \tag{4.11}$$

where the 'relativistic energy' E is defined as

$$E = \frac{\alpha_4}{i}mc^2 \tag{4.12}$$

Combining equation (4.5) with equations (4.8) through (4.12) then yields the fundamental relation

$$E^2 = \pi_r\pi_r c^2 + (mc^2)^2 \tag{4.13}$$

Apparently, the 'rest energy' mc^2 which appears in equation (4.13) is a way of correcting for the fact that $\langle u_r u_r\rangle$, which occurs when considering conservation of translational kinetic energy, differs from $\langle v_r\rangle\langle v_r\rangle$, which occurs after considering conservation of momentum. Note, however, that by using **u** instead of **v** we are ignoring the rotational fluctuations of **v**. These fluctuations seem to carry an additional 'rest energy' of $2mc^2$, and are perhaps related in a fundamental way to 'spin'.

Returning now to the problem of representing explicit time dependence, equation (4.7) implies

$$\frac{\partial\psi}{\partial t} = \alpha_4 c\frac{\partial\psi}{\partial\xi_4} \tag{4.14}$$

This suggests the definition of a 'relativistic time scale'

$$d\tau = \frac{\alpha_4}{i}dt \tag{4.15}$$

so that

$$\frac{\partial\psi}{\partial\tau} = ic\frac{\partial\psi}{\partial\xi_4} \tag{4.16}$$

Using the relativistic time scale, the 'relativistic velocity' is defined as

$$\frac{d\xi_r}{d\tau} = \frac{i}{\alpha_4}\langle v_r\rangle \tag{4.17}$$

Combining equations (4.1), (4.2), (4.4), and (4.17) then yields the Lorentz equation

$$\left(\frac{i}{\alpha_4}\right)^2 = 1 - \frac{d\xi_r\, d\xi_r}{c^2\, d\tau^2} \tag{4.18}$$

Equations (4.9), (4.17) and (4.18) lead finally to the 'relativistic mass' concept through the momentum equation

$$\pi_r = \frac{m}{\sqrt{\left(1 - \frac{d\xi_s\, d\xi_s}{c^2\, d\tau^2}\right)}} \frac{d\xi_r}{d\tau} \tag{4.19}$$

Changes in the Speed of Light

The preceding statistical formalism is valid even if dc/dt is not zero, contrary to the usual requirement expressed by equations (2.18) and (2.20). In a precise sense the condition that dc/dt be zero is equivalent to the adiabatic limit, since, for a particle, the rate of energy transfer as heat is given by (Kornacker, 1968)

$$\frac{dQ}{dt} = m \sum \sigma^2_{v_i, \frac{dv_i}{dt}} \tag{5.1}$$

Therefore, by equations (2.14) and (5.1)

$$\frac{dQ}{dt} = \tfrac{3}{2} m \frac{dc^2}{dt} \tag{5.2}$$

The Einstein work-energy relation

$$dE = d\xi_r \frac{d\pi_r}{d\tau} \tag{5.3}$$

leaves out heat, and its derivation requires constant c. Equation (5.2) shows that these two restrictions are equivalent. Therefore a relativistic theory of processes which include heat (e.g. photon emission) cannot include the constancy of c as an axiom. This result would appear to be especially significant in the fields of high-energy particle physics and cosmology, but such applications have not yet been made.

Reference

Kornacker, K. (1968). *Nature*, **219**, 1283.